Chapter 20 – Profit Maximization

→ Note: Here, we'll deal w/ profit maximization in a competitive market

→ that is firms will take prices for inputs and outputs as given

→ we'll study the firms problem in a non-competitive setting in future chapters, but many situations are well represented by competitive markets and much intuition can be gleaned from these models anyway.

Useful definitions and assumptions

→ Profits : profits = revenue – cost

→ revenue = price × output

→ cost = input ~~costs~~ prices × input quantities

→ Costs → these include actual and opportunity costs

→ Timing: revenues are a flow (e.g. \$'s per year)

→ want to measure all things on the same scale

→ so put inputs in terms of flows

→ e.g. hours and dollars per hour for labor input, rental rate per hour use capital (a stock)

→ Accounting vs. economic profits

→ b/c the economic costs include actual and opportunity costs, economic profits will differ from accounting profits

→ accounting profits don't include opportunity costs

→ e.g. lets say you open a business where you need to put up $100 and you make 5% return (a $5 accounting profit)

→ Great, right? Well what else could you do w/ that $100? Maybe invest in stock mkt and get 6% return.

→ In that case, economic profit is -$1, a -1% return.

→ Assumption: Firms maximize profits.

→ If consider firms over multiple periods and they face uncertain shocks, profits aren't the right measure

→ These firms maximize shareholder value

Profit maximization

→ we'll define 2 periods:

1) Short run: at least one factor of production fixed

2) Long run: all factors of production variable

⇒ Short Run Profit Maximization

→ The problem:

$$\max_{x_1} \; p f(x_1, \bar{x}_2) - w_1 x_1 - w_2 \bar{x}_2$$

→ "bar" over x_2 means it's fixed

→ w_1, w_2 are the per unit costs/prices for using x_1 and x_2, respectively

~~→ you might thin~~

→ want to find optimal x_1

→ The optimal x_1 is that which maximizes the profit function

⇒ slope of profit function at x_1^* must be zero.

→ so we find that condition

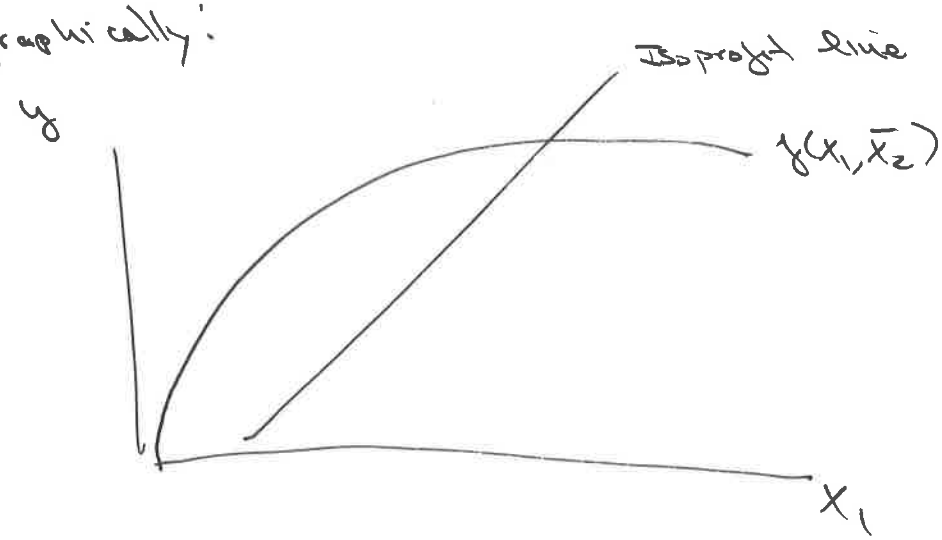
$$\underbrace{\pi}_{\text{profit}} = p f(x_1, \bar{x}_2) - w_1 x_1 - w_2 \bar{x}_2$$

$$\frac{\partial \pi}{\partial x_1} = \underbrace{p \frac{\partial f(x_1, \bar{x}_2)}{\partial x_1} - w_1} = 0$$

$$\Rightarrow \underbrace{p \frac{\partial f(x_1, \bar{x}_2)}{\partial x_1}}_{} = w_1$$

the x_1 that solves this is x_1^*, the optimal x_1

→ one can also think about this solution graphically:



Isoprofit line: a line along which all combinations of the input goods and output good give constant profit

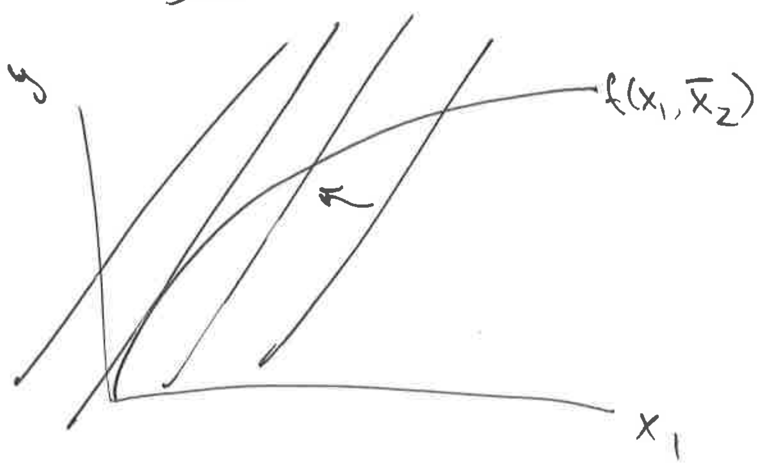
⇒ take $\underbrace{\pi = py - w_1 x_1 - w_2 \bar{x}_2}$

solve for y

$$\Rightarrow y = \frac{\pi + w_1 x_1 + w_2 \bar{x}_2}{p}$$

this is the equation for the isoprofit line

⇒ if max profits is the goal, want to be on the highest isoprofit line:



→ but must be on feasible isoprofit line → i.e. it must be in the production set

⇒ want isoprofit line tangent to production function

→ tangent ⇒ slopes equal

→ slope of isoprofit $= \frac{w_1}{p}$

→ slope of production function $= MP_1$

$\Rightarrow \frac{w_1}{p} = MP_1$

$\Rightarrow w_1 = MP_1 p = p \frac{\partial f(x_1, \bar{x}_2)}{\partial x_1}$ → same as answer derive before

Example of short run π-max

→ Shoe Shine

→ Rent ~~stand~~ chair → pre paid rent, so it's fixed

→ $f(x_1, x_2) = f(\text{hours}, \#\cancel{\text{stands}}\ \text{chairs}) = \#$ shoes shined

$= 12 * \text{hours}^{1/2}\ \#\text{chairs}^{1/2}$

→ price = \$3 per shoe shined

→ wage = \$6 . chair rental cost per ~~hour~~ per chair = \$10

→ chairs fixed at 1

$$\Rightarrow \pi = 3\,(12\,\text{hours}^{1/2}\ 1^{1/2}) - (6 * \text{hours}) - (10 \times 1)$$

$$= 3 \times 12 * \text{hours}^{1/2} - 6\,\text{hours} - 10$$

$$\frac{\partial \pi}{\partial \text{hours}} = \frac{1}{2}\,36\ \text{hours}^{-1/2} - 6 = 0$$

$$\Rightarrow 18\ \text{hours}^{-1/2} = 6$$

$$\text{hours}^{-1/2} = \frac{6}{18}$$

$$\text{hours}^{-1/2} = 1/3$$

$$\text{hours} = (1/3)^{-2} = 3^2$$

$$\underline{\text{hours} = 9}$$

Comparative Statics

→ comparative statics can help us understand how the endogenous variables change w/ a change in exogenous variables/parameters

→ e.g. we can ask how a factor demand would change if its price change.

→ Shoe shine:

→ what if outside option for his work goes up from \$6 to \$10

→ how does hours shining shoes respond?

$$\pi = 36\, hours^{1/2} - 10 \times hours - 10$$

$$\Rightarrow \frac{\partial \pi}{\partial hours} = \tfrac{1}{2} 36\, hours^{-1/2} = 10$$

$$18\, hours^{-1/2} = 10$$

$$hours^{-1/2} = \frac{10}{18}$$

$$hours^{-1/2} = 5/9$$

$$hours = \left(\frac{9}{5}\right)^2 = \underline{3.24}$$

~~Analytically~~, We can solve for such a response more generally:

The First order Condition is:

$$p f'(x^*) = w$$

The Second order condition is:

$$f''(x^*) \leq 0$$

slope declining at optimum if max

→ the factor demand, $x(w)$ is ~~the~~ given by the function that satisfies these 2 conditions for all w

e.g. let $f(x) = x^{\frac{1}{2}}$

FOC: $p \frac{1}{2} x^{-\frac{1}{2}} = w$

$$\Rightarrow 2\frac{w}{p} = x^{-\frac{1}{2}}$$

$$\Rightarrow x = \left(\frac{2w}{p}\right)^{-2}$$

$$x = \left(\frac{p}{2w}\right)^{2} = x(w)$$

→ this factor demand must solve the FOC ~~and~~ ~~2nd order cond~~: (actually, it solves it by definition):

$$\underbrace{f'(x(w)) = w}_{\text{differentiate w.r.t. } w}$$

$$f''(x(w)) \frac{\partial x(w)}{\partial w} = 1$$

$$\Rightarrow \frac{\partial x(w)}{\partial (w)} = \frac{1}{\underbrace{f''(x(w))}_{\leq 0}}$$

(see 2nd order condition

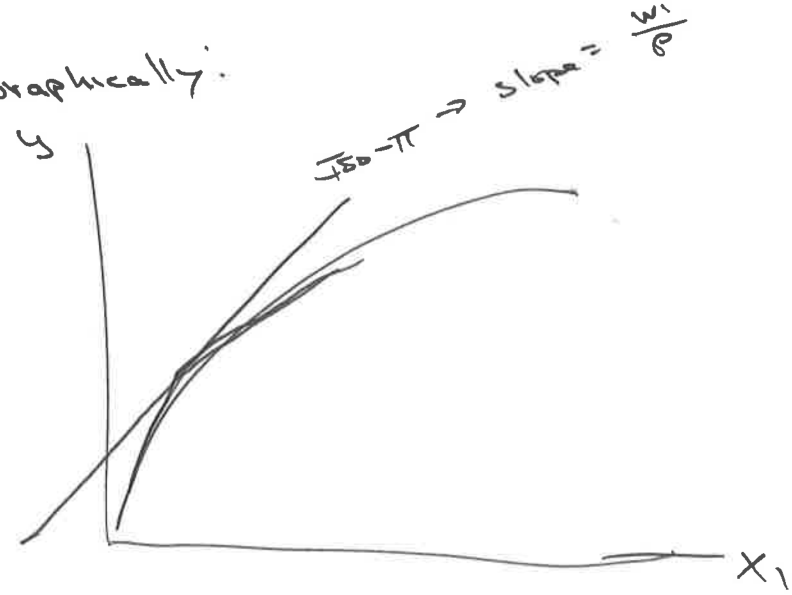
→ so we know

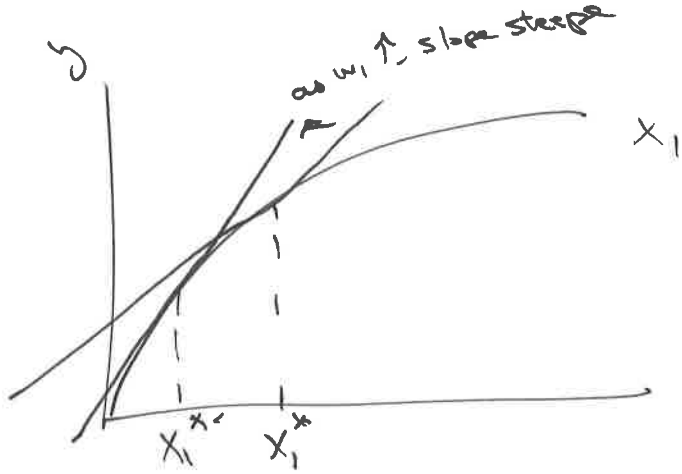
$$\frac{\partial x(w)}{\partial w} \leq 0$$

→ factor demands do not rise w/ increases in price of that factor

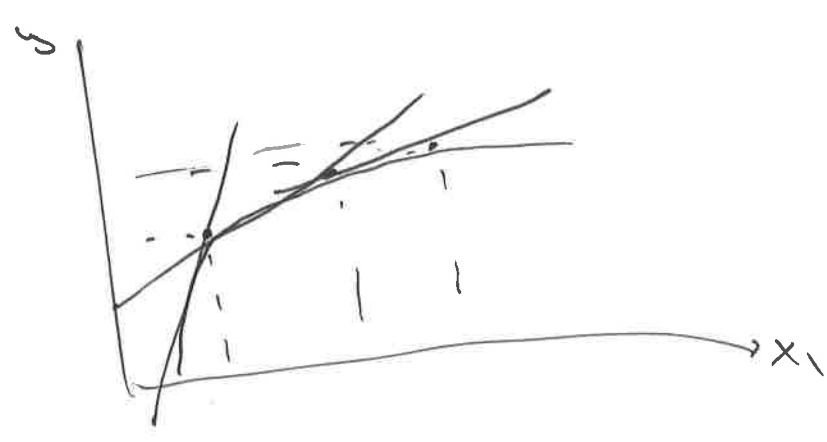
⇒ factor demand function slopes down

Graphically:





→ Note that this also means if observe (x_1, y) at diff factor prices w_1 can trace out prod. function!



Profit-max in the Long Run

→ Long run ⇒ all inputs variable

→ The problem:

$$\max_{x_1, x_2} \; p f(x_1, x_2) - w_1 x_1 - w_2 x_2$$

→ Now 2 FOCs

1) $p \dfrac{\partial f(x_1, x_2)}{\partial x_1} - w_1 = 0$

2) $p \dfrac{\partial f(x_1, x_2)}{\partial x_2} - w_2 = 0$

⇒ $p\, MP_1 = w_1$

→ $p\, MP_2 = w_2$

we have 2 equations to solve for x_1, x_2

e.g. $f(x_1, x_2) = x_1^{a} x_2^{1-a}$

⇒ $p\, MP_1 = w_1$

$p\, a x_1^{a-1} x_2^{1-a} = w_1$

and

$p\, MP_2 = w_2$

$p (1-a) x_1^{a} x_2^{-a} = w_2$

$$\frac{\cancel{p}\, a x_1^{\alpha-1} \cancel{x_2}^{1-\alpha}}{\cancel{p}(1-a)\cancel{x_1}^{\alpha} \cancel{x_2}^{-\alpha}} = \frac{w_1}{w_2}$$

$$\frac{a x_2}{(1-a) x_1} = \frac{w_1}{w_2}$$

$$\underbrace{x_2 = \frac{w_1}{w_2} \cdot \frac{(1-a)}{a}}\, x_1$$

note, can't pin down x_2, x_1 by self - only ratio

→ this is b/c constant returns to scale and ~~compet~~ competitive eq'm

→ firm has ~~zero~~ zero π at any level of output

→ for a given $y = f(x_1, x_2)$ we have

FOC 1) $\Rightarrow p \frac{a y}{x_1} = w_1$

2) $\Rightarrow p \frac{(1-a) y}{x_2} = w_2$

$1 + 2 \Rightarrow x_1 = \frac{p a y}{w_1}$ ✓

$x_2 = \frac{p(1-a) y}{w_2}$ –

Profit Maximization and Returns to Scale

→ profits:

$$\pi = py - w_1 x_1 - w_2 x_2$$

$$= pf(x_1, x_2) - w_1 x_1 - w_2 x_2$$

→ if CRS:

$$\pi' = pf(zx_1, zx_2) - w_1 z x_1 - w_2 z x_2$$

$$= zpf(x_1, x_2) - z(w_1 x_1 + w_2 x_2)$$

$$= z[pf(x_1, x_2) - w_1 x_1 - w_2 x_2]$$

$$= z\pi$$

⇒ if increase inputs by a factor z, ~~outputs~~ π increase by factor z

→ but then, if $\pi > 0$, the optimal firm size (that maximizes profits) is infinite.

→ this is inconsistent w/ ~~the~~ competitive mkt assumption

→ so if firm has CRS, $\pi = 0$ if market competitive and firms maximizing profits

Inverse factor demand curves

→ Just ~~as~~ as we did for consumers, we can solve for price that corresponds to each quantity of factor demanded.

→ These functions are called the **inverse factor demand functions**.

→ recall FOC:

$$p\,MP_1(x_1, \bar{x}_2) = w_1$$

→ Inverse factor demand: $w_1 = p\,MP_1(x_1, x_2)$

→ for a fixed x_2 (could be LR optimum)